

Atmospheric carbon dioxide levels for the last 500 million years

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ABSTRACT

The last 500 million years of the strontium-isotope record are shown to correlate significantly with the concurrent record of isotopic fractionation between inorganic and organic carbon after the effects of recycled sediment are removed from the strontium signal. The correlation is shown to result from the common dependence of both signals on weathering and magmatic processes. Because the long-term evolution of carbon dioxide levels depends similarly on weathering and magmatism, the relative fluctuations of CO₂ levels are inferred from the shared fluctuations of the isotopic records. The resulting CO₂ signal exhibits no systematic correspondence with the geologic record of climatic variations at tectonic time scales.

The long-term carbon cycle is controlled by chemical weathering, volcanic and metamorphic degassing, and the burial of organic carbon [1, 2]. Ancient atmospheric carbon dioxide levels are reflected in the isotopic content of organic carbon [3] and, less directly, strontium [4] in marine sedimentary rocks; the former because photosynthetic carbon isotope fractionation is sensitive to CO₂ levels, and the latter because weathering and degassing are associated with extreme values of the abundance ratio ⁸⁷Sr/⁸⁶Sr. However, attempts to use these geochemical signals to estimate past CO₂ levels [5–8] are hindered by the signals' additional relationships to various tectonic [9, 10] and biological [11] effects. Moreover the strontium signal has proven especially difficult to parse [12–15].

Here I attempt to resolve these ambiguities in the isotopic signals of carbon and strontium. First, it is shown that the last 500 million years of the strontium signal, after transformation to remove the effects of recycled sediment [16, 17], correlate significantly with the concurrent record of isotopic fractionation between inorganic and organic carbon [3]. This empirical result is supplemented by the theoretical deduction that the two records are linked by their common dependence on rates of continental weathering and magmatic activity. The assumption that CO₂ levels fall with the former and rise with the latter then indicates that an appropriate average of the two records should reflect the long-term fluctuations of the partial pressure of atmospheric CO₂. The CO₂ signal derived from this analysis represents fluctuations at time scales greater than about 10 My. Comparison with the geologic record of climatic variations [18] reveals no obvious correspondence.

Strontium and carbon isotopic signals

Figure 1 shows the strontium and carbon isotopic signals for the last 500 million years (My). The data for the strontium isotope ratios $^{87}\text{Sr}/^{86}\text{Sr}$ were compiled by Veizer et al. [4] and Walter et al. [19]; the former source accounts for all $^{87}\text{Sr}/^{86}\text{Sr}$ data younger than 520 My, whereas the latter was used for data extending back to 608 My (not shown). Both sets of $^{87}\text{Sr}/^{86}\text{Sr}$ data were first averaged in non-overlapping time windows of 10 My. The resulting unevenly spaced record was then transformed [20] to an evenly spaced record with time increments of approximately 10 My and no contributions to the power spectral density at periods less than 21 My.

The second record in Figure 1, compiled by Hayes et al. [3], derives from the isotopic composition of marine organic carbon and carbonate carbon. From isotopic abundance ratios $R_x = (^{13}\text{C}/^{12}\text{C})_x$ for carbon in sample x , the isotopic fractionation between sample x and a standard (STD) sample, $\delta_x = 1000[(R_x - R_{\text{STD}})/R_{\text{STD}}]$, is obtained for carbonate (δ_a) and organic (δ_o) carbon. The isotopic fractionation ε_{toc} between total organic carbon and sedimentary carbonates is then given approximately by $\varepsilon_{\text{toc}} = \delta_a - \delta_o$, which is the second signal plotted in Figure 1.

Figure 1 shows a surprising similarity between the two records for fluctuations with periods less than about 100 My. However the correlation between the two time series is not statistically significant* (Spearman rank correlation coefficient [21] $R_s = -0.40$, $P = 0.17$, $N = 46$) because the longer-period fluctuations are not in phase. It is therefore interesting to ask why the two records appear similar at shorter time scales and dissimilar at longer time scales.

*Correlations between $\varepsilon_{\text{toc}}(t)$ and $s(t)$ or $g(t)$ are computed from the N equal-time pairs obtained after linearly interpolating the Sr signal so that it is sampled at the same times as ε_{toc} . The statistical significance P is one-sided and was estimated using the Monte Carlo technique described in Ref. [8].

Recent work has shown that the measurements of ε_{toc} approximately fit the empirical relation [3, 22]

$$\varepsilon_{\text{toc}} = \varepsilon_0 - \Gamma/\Phi. \quad (1)$$

The parameter ε_0 represents the isotopic effects of photosynthesis and secondary biological processes along with the isotopic depletion of dissolved CO_2 in surface waters relative to sedimentary carbonate. Because ε_0 is approximately constant throughout the period plotted in Figure 1, the main source of ε_{toc} 's fluctuations are contained in isotopic effects due to changing algal physiology—the permeability, surface-to-volume ratio, and growth rate of algal cells—represented by Γ , and the concentration Φ of dissolved carbon dioxide in surface waters [3, 8].

Although the mechanisms responsible for the fluctuations of $s = {}^{87}\text{Sr}/{}^{86}\text{Sr}$ are subject to much debate [4, 9, 10, 12–15], some aspects of the signal's evolution are nevertheless clear. Since ${}^{87}\text{Rb}$ decays to ${}^{87}\text{Sr}$ with a half-life of ~ 48 billion years, the supply of ${}^{87}\text{Sr}$ may be taken to be approximately constant over the last 500 My. However it is not uniformly distributed: the fluvial input to the oceans is derived in part from rocks—both silicates and metamorphosed carbonates [12–15]—that are relatively enriched in radiogenic Sr ($s \simeq 0.712$ or greater) compared to the non-radiogenic Sr of mantle origin supplied at hydrothermal vents ($s \simeq 0.7035$) [23]. The value of $s(t)$ at some particular time t represents, to first approximation, the relative fluxes of these two extreme values.

However this first approximation ignores the recycling of rocks in the sedimentary cycle [16]. As pointed out by Brass [17], about 75% of the strontium input to the oceans should come from the weathering of exposed carbonates of marine origin. Because these carbonates retain a memory of s at the time of deposition, their contribution to sedimentary Sr significantly damps the signal coming from hydrothermal

vents (low s) and rocks containing radiogenic Sr (high s) [9].

Denoting the fluvial flux of radiogenic Sr by r , the input from hydrothermal vents by v_h , and the memory effect by m , a simple expression for the strontium isotope ratio s is

$$s = \mu m + (1 - \mu) g(r, v_h) \quad (2)$$

where $0 \leq \mu \leq 1$ is the fraction of sedimentary Sr deriving from the memory flux and g is a function that depends on both r and v_h . The memory term can be approximated by assuming that sedimentary strontium is weathered at a rate proportional to its mass [16]. Then $m(t)$ is simply a weighted (exponentially decaying) average of $s(\tau)$, $\tau < t$ [17], which we express by the convolution

$$m(t) = \int_{-\infty}^t \lambda e^{-\lambda(t-\tau)} s(\tau) d\tau. \quad (3)$$

The parameter λ is related to the half-life $t_{1/2}$ of sedimentary Sr; i.e., $\lambda = -(\ln 2)/t_{1/2}$. Brass estimates $57 \text{ My} < t_{1/2} < 102 \text{ My}$ [17]. Therefore the memory effect should strongly influence the fluctuations of s at long time scales (greater than about 100 My), whereas the short time scale fluctuations should be relatively unaffected.

The short-time correlations may be explained in part by noting the common dependence of ε_{toc} and g on the global weathering rate w (i.e., the flux of all weathered products from the continents to the oceans) and the rate of magmatic activity v associated not only with the hydrothermal flux v_h but also volcanic degassing. Assuming that $r = r(w)$ and $v_h = v_h(v)$, g 's dependence on w and v may be written[†]

$$g = g(r(w), v_h(v)). \quad (4)$$

With respect to ε_{toc} , the physiological term Γ may likewise depend on w because of changing nutrient concentrations in the oceans. Γ could also depend on CO₂ levels,

[†]Although w and v could conceivably depend on one another, here such dependencies are assumed to be insignificant compared to any independent variations.

which in turn depend on degassing, weathering rates—because of the net uptake rate $u = u(w)$ of atmospheric CO₂ associated with silicate weathering [1, 2, 24]—and other processes, such as organic carbon burial, which we collectively designate by b . Aggregating these dependencies then yields

$$\varepsilon_{\text{toc}} = \varepsilon_0 - \frac{\Gamma\left(w, \Phi\left(u(w), v, b\right)\right)}{\Phi\left(u(w), v, b\right)}, \quad (5)$$

Comparison of equations (4) and (5) directly reveals the joint dependence of g and ε_{toc} on v and w .

Removal of the effects of sedimentary recycling

Because the strontium signal s contains a memory effect whereas ε_{toc} does not, removal of the memory effect should reveal correlations between s and ε_{toc} at both short and long time scales, thereby confirming the joint dependence on v and w . To test this hypothesis, I first assume it is true and use it to estimate g . For a given λ and μ , m is calculated by discretizing equation (3) for $\tau > -608$ My. Equation (2) is then solved for g :

$$g_{\lambda\mu} = \frac{s - \mu m_{\lambda}}{1 - \mu}. \quad (6)$$

Here the subscripts make explicit the dependencies on λ and μ .

Let $R_{\lambda\mu}(\varepsilon_{\text{toc}}, g_{\lambda\mu})$ be the Pearson product-moment correlation coefficient [21] that quantifies the similarity of ε_{toc} and $g_{\lambda\mu}$. From the foregoing argument, the best estimate of λ and μ should be that which minimizes $R_{\lambda\mu}$ (because the expected correlation is negative). Figure 2 shows contours of R as a function of μ and $t_{1/2} = -(\ln 2)/\lambda$. The minimum $R_{\lambda\mu} = -0.81$ occurs at $t_{1/2} = 54$ My and $\mu = 0.99$. However the corresponding estimate of $g_{\lambda\mu}$ includes values below the hydrothermal minimum of 0.7035. Such non-physical ratios probably derive from the amplification of any

measurement noise resulting from division by the small quantity $1 - \mu$ in equation (6). Indeed, the dark gray region in Figure 2 indicates that all such results only occur for μ close to unity. I therefore define the best estimates λ^* , μ^* of λ , μ by minimizing $R_{\lambda\mu}$ subject to the constraint that $g_{\lambda\mu} > 0.7035$. One then finds $t_{1/2}^* = -\ln 2/\lambda^* = 41$ My and $\mu^* = 0.83$, corresponding to $R_{\lambda^*\mu^*} = -0.80$ (within 1% of the unconstrained minimum). These results are in reasonable accord with Brass’s estimate of the half-life of sedimentary Sr (57–102 My) [17] and his conclusion that “strontium leached from limestones is about 75% of the total input” [17].

Figure 3 shows $g_{\lambda^*\mu^*}$ compared to ε_{toc} . Compared with Figure 1, one sees that both the long and short period fluctuations are now not only approximately equally correlated, but the correlation is also significant ($R_s = -0.74$, $P < 10^{-3}$, $N = 46$). The hypothesis that recycled sediment partially obscured an inherent correlation due to a shared dependence on weathering and volcanic processes is therefore confirmed.

Relationship to CO₂ levels

To further understand the correlation I make the following assumptions concerning the functional dependencies contained in equations (4) and (5):

$$dr/dw > 0, \quad du/dw > 0, \quad \text{and} \quad dv_h/dv > 0. \quad (7)$$

The first two state that the flux of radiogenic Sr to the oceans and the uptake of atmospheric CO₂ due to silicate weathering increase as the global weathering rate (of all minerals in all terranes) increases. The third formalizes the natural assumption that the hydrothermal flux of Sr varies with the same sign as all magmatic processes.

Because the Sr isotope ratios increase with increasing r (i.e., $\partial g/\partial r > 0$) and decrease with increasing v_h (i.e., $\partial g/\partial v_h < 0$), one has

$$\frac{\partial g}{\partial v} = \frac{\partial g}{\partial v_h} \frac{dv_h}{dv} < 0 \quad \text{and} \quad \frac{\partial g}{\partial w} = \frac{\partial g}{\partial r} \frac{dr}{dw} > 0. \quad (8)$$

Since g and ε_{toc} are negatively correlated, one expects that ε_{toc} depends on v and w in the opposite sense:

$$\frac{\partial \varepsilon_{\text{toc}}}{\partial v} > 0 \quad \text{and} \quad \frac{\partial \varepsilon_{\text{toc}}}{\partial w} < 0. \quad (9)$$

The inequalities (8) and (9) have been obtained without making any explicit assumption concerning any dependence of r on u nor Γ on v and w . Because they show that v and w each influence g and ε_{toc} with opposite signs, the relative fluctuations of any other quantity that also depends on v and w with opposite signs may be inferred from the shared fluctuations of g and ε_{toc} .[‡] Specifically, the concentration Φ of oceanic CO₂ responds positively to v while its response to weathering is negative:

$$\frac{\partial \Phi}{\partial v} > 0 \quad \text{and} \quad \frac{\partial \Phi}{\partial w} = \frac{\partial \Phi}{\partial u} \frac{du}{dw} < 0. \quad (10)$$

Thus the shared fluctuations of g and ε_{toc} indicate the relative fluctuations of ancient CO₂ levels.[§]

Estimation of the CO₂ signal

I proceed to estimate the CO₂ signal explicitly. First, the highest measurement of ε_{toc} , at -175 My, is dropped because of its statistical insignificance [3]. I then transform $g(t) \rightarrow \varepsilon_g(t)$, a strontium-derived estimate of ε_{toc} , by mapping the left axis of Figure 3 to the corresponding value on the right axis. The quantities ε_{toc} and ε_g

[‡]One also requires that if changes in v or w dominate one process then they dominate all processes.

[§]Because Φ can also depend on other processes b such as organic carbon burial, the shared fluctuations of g and ε_{toc} do not necessarily reflect the full evolution of Φ , but rather the “partial” contribution of v and w alone. However, the good correlation of Figure 3 indicates that b ’s influence on Φ ’s fluctuations has been small, except possibly in the Carboniferous. Thus one may conclude that the fluctuations of g and ε_{toc} follow to good approximation the fluctuations of Φ with opposite signs.

are then averaged to obtain

$$\zeta(t_i) = [\varepsilon_g(t_i) + \varepsilon_{\text{toc}}(t_i)] / 2. \quad (11)$$

The times t_i are given by the points where ε_{toc} is specified and $\varepsilon_g(t_i)$ is obtained by linear interpolation. Here it is implicitly assumed that ε_{toc} and ε_g contain a common signal that is enhanced by averaging. Statistical arguments suggest that ζ 's root-mean-square signal-to-noise ratio lies between about 2 and 3.[¶]

Assume now that Γ and ε_0 are constant and define the dimensionless CO₂ concentration $\phi = \Phi \varepsilon_0 / \Gamma$ [8]. In terms of ϕ and ζ , equation (1) then yields

$$\phi(t) = \frac{\varepsilon_0}{\varepsilon_0 - \zeta(t)}. \quad (12)$$

At long time scales such that the oceans are in equilibrium with the atmosphere, the partial pressure $p\text{CO}_2$ is proportional to ϕ [1]. The relative fluctuations of $p\text{CO}_2$ are therefore given by $\phi(t)/\phi(0)$, which is plotted in Figure 4 for $\varepsilon_0 = 36$ per mil (‰). The gray area surrounding the $p\text{CO}_2$ curve in Figure 4 brackets this result for $\varepsilon_0 = 35\%$ and 38% , a range consistent with previous estimates [3].

Figure 4 reveals that CO₂ levels have mostly decreased for the last 175 My. Prior to that point they appear to have fluctuated from about two to four times modern

[¶]To estimate the signal-to-noise ratio of the time series $\zeta(t)$ defined by equation (11), assume that ε_{toc} and ε_g are each composed of a shared but unknown signal z in the presence of zero-mean noise η_{toc} and η_g , respectively: $\varepsilon_{\text{toc}} = z + \eta_{\text{toc}}$ and $\varepsilon_g = z + \eta_g$. The quantity ζ is an estimate of z . Its signal-to-noise ratio may be computed under the assumption that η_{toc} and η_g each have variance σ_η^2 , z has variance σ_z^2 , and η_{toc} , η_g , and z are each uncorrelated to the other. The expectation of the correlation coefficient R is then $\sigma_z^2 / (\sigma_z^2 + \sigma_\eta^2)$ [25]. The observed correlation $|R| = 0.80$ then yields $\sigma_z^2 / \sigma_\eta^2 = 4$, or that the root-mean-square (rms) signal fluctuation is twice that of the noise. The rms signal-to-noise ratio of ζ can be as large as a factor of $\sqrt{2}$ greater, i.e., it should range between about 2 and 3.

levels with a dominant period of about 100 My. The decline for the last 175 My is also present in several previous $p\text{CO}_2$ reconstructions [7,8,26,27] and the entire curve displays some similarity to a previous estimate derived from the geologic record of carbonate formation [26]. Although the period before -175 My differs substantially from previous geochemical model calculations [7], an approximate error estimate lends considerable credence to the $p\text{CO}_2$ curve of Figure 4. Specifically, ϕ should inherit ζ 's signal-to-noise ratio of 2–3. This correspondence would be exact if $\phi(\zeta)$ were linear. Because $\phi(\zeta)$ is monotonic for the observed range of ζ , its nonlinearity does not affect the timing of the maxima and minima of the $p\text{CO}_2$ curve. Thus the linear error estimate remains pertinent.

Comparison with the climate record

Using a variety of sedimentological criteria, Frakes et al. [18] have concluded that the Earth's climate has cycled several times between warm and cool modes for roughly the last 600 My. Recent work by Veizer et al. [28], based on measurements of oxygen isotopes in calcite and aragonite shells, appears to confirm the existence of these long-period (~ 135 My) climatic fluctuations. Changes in CO_2 levels are usually assumed to be among the dominant mechanisms driving such long-term climate change [29].

It is therefore interesting to ask what, if any, correspondence exists between ancient climate and the estimate of $p\text{CO}_2$ in Figure 4. The gray bars at the top of Figure 4 correspond to the periods when the global climate was cool; the intervening white space corresponds to the warm modes [18]. The most recent cool period corresponds to relatively low CO_2 levels, as is widely expected [30]. However no correspondence between $p\text{CO}_2$ and climate is evident in the remainder of the record, in part because the apparent 100 My cycle of the $p\text{CO}_2$ record does not match to the longer climatic

cycle. The lack of correlation remains if one calculates the change in average global surface temperature resulting from changes in $p\text{CO}_2$ and the solar constant using energy-balance arguments [7, 26].

Superficially, this observation would seem to imply that $p\text{CO}_2$ does not exert dominant control on the Earth’s climate at time scales greater than about 10 My. A wealth of evidence, however, suggests that $p\text{CO}_2$ exerts at least *some* control (see Crowley and Berner [30] for a recent review). Figure 4 cannot by itself refute this assumption. Instead it simply shows that the “null hypothesis” that $p\text{CO}_2$ and climate are unrelated cannot be rejected on the basis of this evidence alone.

Discussion and conclusion

One of the principal contributions of this study is methodological. From observations of a weak correlation between strontium and carbon isotopic signals (Figure 1) and their shared dependence on global weathering rates and magmatic activity, weathering and magmatism are deduced to be the main processes driving the signals’ fluctuations. Correction for the effects of sedimentary recycling enhances the correlation (Figure 3), indicates a strong shared signal, and strengthens this conclusion.

A second, crucial, step is to note that any quantity with a similar joint dependence on weathering and magmatic processes may be expected to display similar fluctuations. Here attention has been focused on CO_2 levels; as for the strontium and carbon isotopic signals, CO_2 levels depend on weathering and magmatism with opposite signs, and should therefore fluctuate roughly in sync with the isotopic signals. Because the reasoning is general, it need not be limited to CO_2 . Among the many possible applications, the case of oceanic phosphate concentrations is particularly interesting. Phosphate concentrations should increase with weathering and decrease

with hydrothermal activity [31]; thus the methodology in this paper may be applicable to their reconstruction. Moreover, since phosphorus is a limiting nutrient, oceanic productivity may be expected to covary positively with its concentration in seawater. This suggests that CO₂ levels and productivity covary negatively at geologic time scales [8].

Such reasoning naturally raises the issue of cause and effect. This study indicates that degassing and silicate weathering were the primary controls on the carbon cycle for the last 500 My. But the results do not themselves indicate whether either of these mechanisms dominated, or whether weathering was driven by the diversification of land plants [8], continental collisions [9], or a complex combination of tectonic, biological, and geochemical processes [7]. They do however offer a new view of the long-term fluctuations of $p\text{CO}_2$ that will hopefully stimulate novel approaches to the study of biogeochemical cycles at evolutionary time scales.

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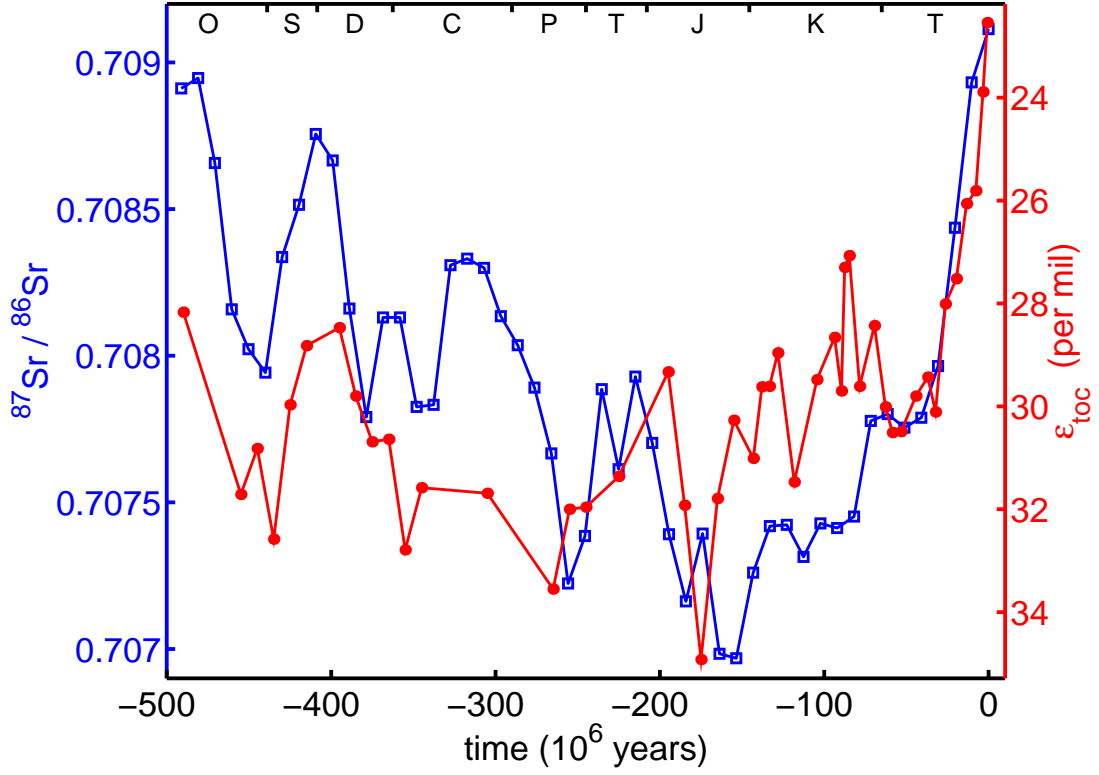


FIG. 1. Data for ϵ_{toc} (red filled circles) [3] and $^{87}\text{Sr}/^{86}\text{Sr}$ (blue open squares) [4]. The time scale for ϵ_{toc} has been revised from the original to match the the scheme [32] used for the strontium data. The capital letters (except for K) correspond to the following geologic periods: Ordovician, Silurian, Devonian, Carboniferous, Permian, Triassic, Jurassic, Cretaceous, and Tertiary.

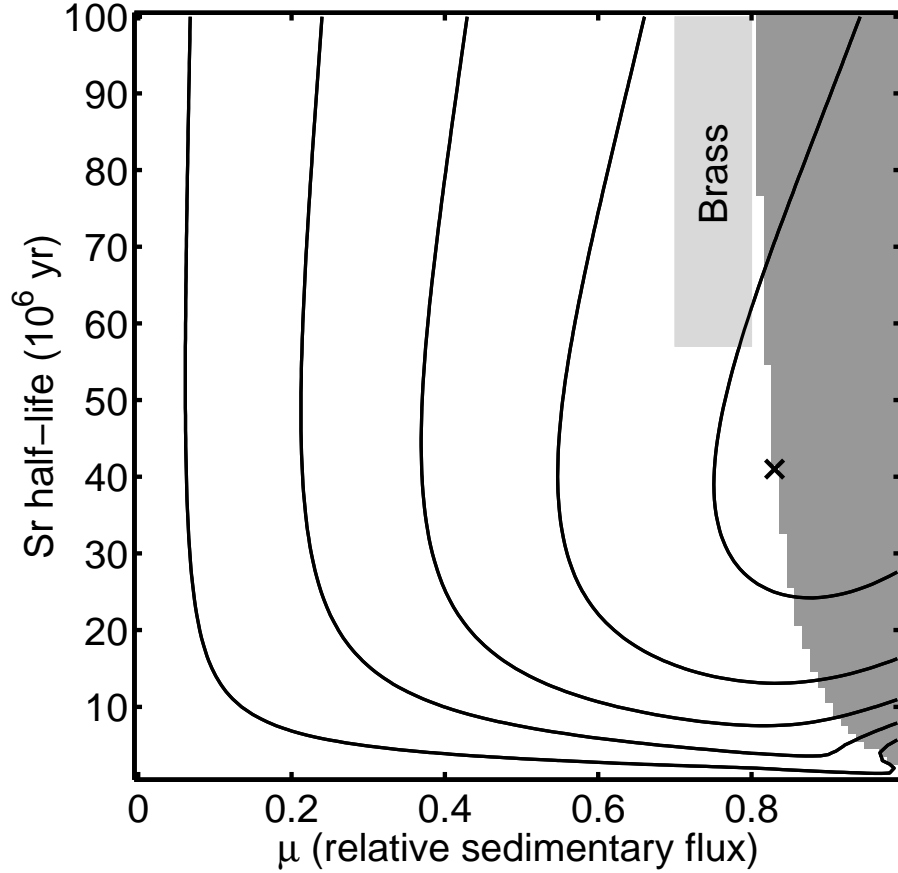


FIG. 2. Contours of $R_{\lambda\mu}$ as a function of μ , the fraction of sedimentary Sr deriving from the memory flux, and $t_{1/2} = -(\ln 2)/\lambda$, the half-life of sedimentary Sr. R was calculated for $\mu = 0, 0.01, \dots, 0.99$ and $t_{1/2} = 1, 2, \dots, 99$ My. For large half-life the contours decrease from left to right as follow: $-0.60, -0.65, -0.70, -0.75, -0.79$. The symbol \times marks the minimum, $R_{\lambda^*\mu^*} = -0.80$, obtained under the constraint that $g(t) > 0.7035$. The area shaded dark gray on the right does not satisfy the constraint. The rectangular area labeled “Brass” corresponds to previous estimates obtained by geochemical arguments [17]; its horizontal extent has not been explicitly computed.

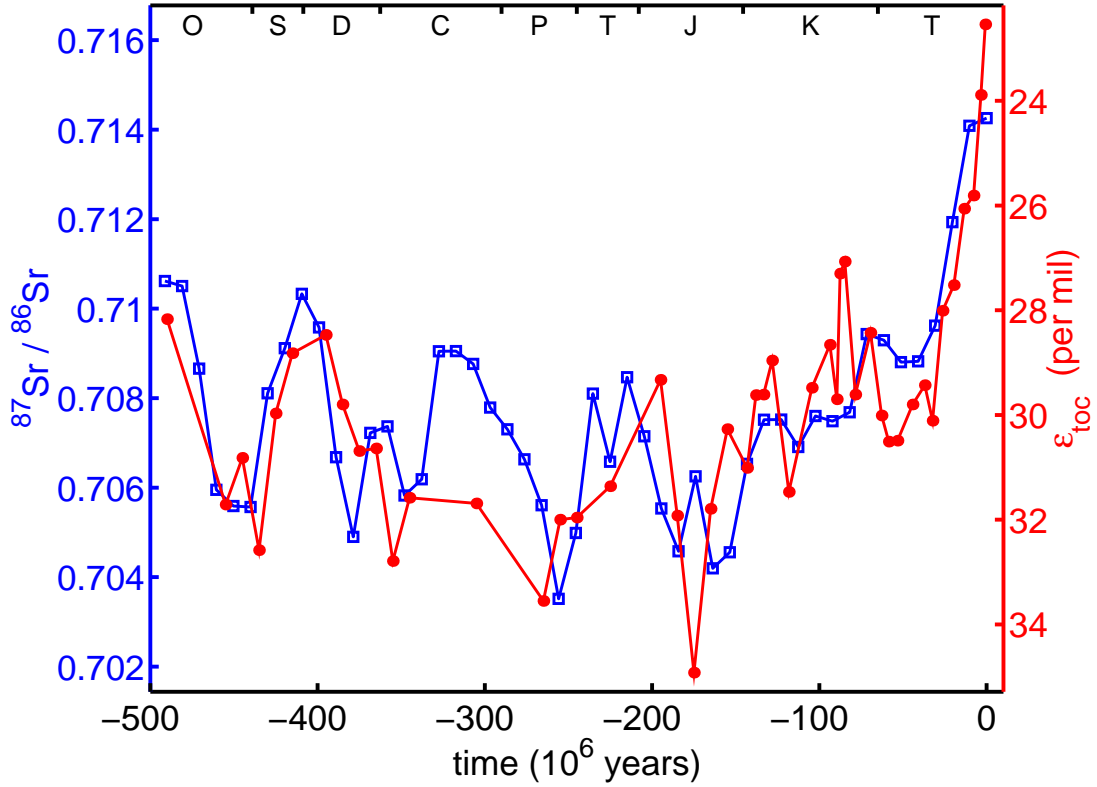


FIG. 3. The function g (blue squares) obtained by removing the memory flux from the $^{87}\text{Sr}/^{86}\text{Sr}$ data of Figure 1, along with ϵ_{toc} (red circles). Note that the range of the $^{87}\text{Sr}/^{86}\text{Sr}$ curve is approximately five times greater than in Figure 1. The data are plotted such that the mean of both time series lies on the same horizontal line and their root-mean-square fluctuations have the same vertical extent.

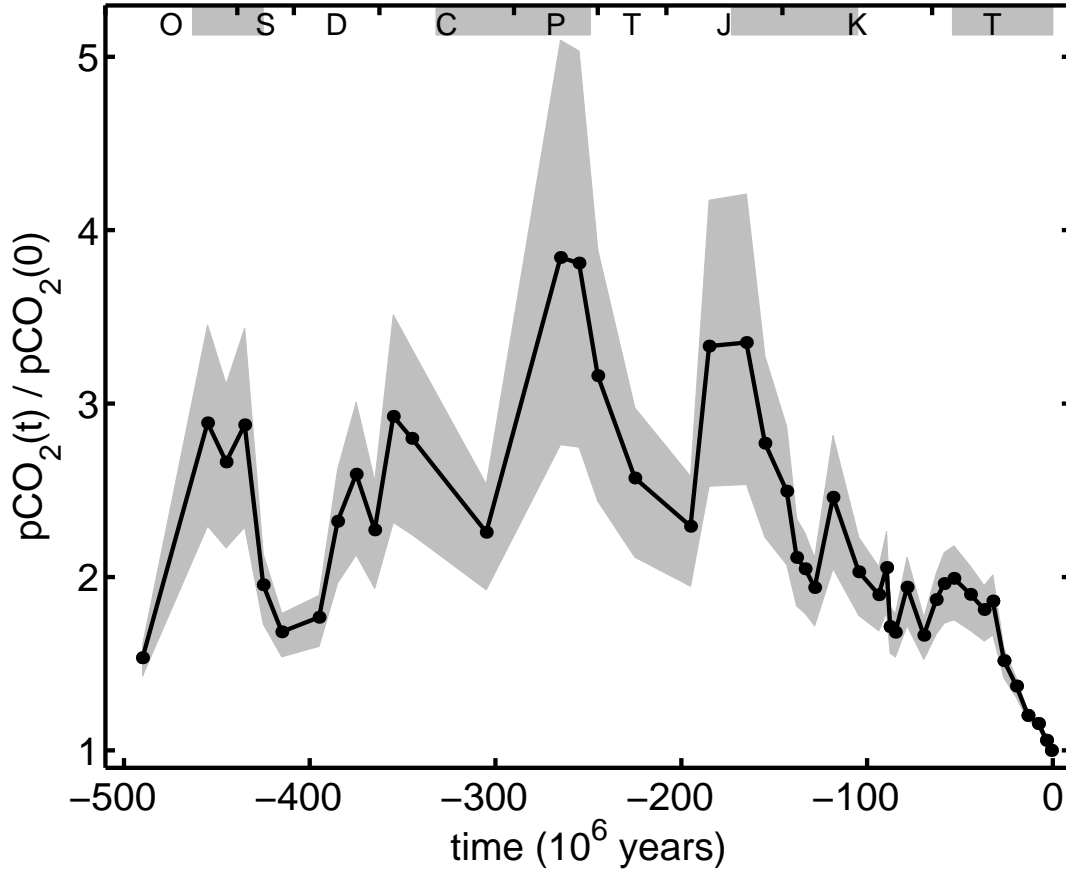


FIG. 4. Fluctuations of $p\text{CO}_2$ for the last 500 million years, normalized by the estimate of $p\text{CO}_2$ obtained from the most recent value of ζ . The solid line is obtained from equation (12) using $\varepsilon_0 = 36\%$. The lower and upper limits of the gray area surrounding the $p\text{CO}_2$ curve result from $\varepsilon_0 = 38\%$ and 35% , respectively. The gray bars at the top correspond to periods when the Earth's climate was relatively cool; the white spaces between them correspond to warm modes [18].